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OBSERVATION ALGORITHM FOR ESSENTIALLY NONLINEAR MOBILE ROBOTS

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Abstract. *This paper is about a new approach of integration of the non-linear observer based on Luenberger, backstepping and disturbance approximation with Unscented Kalman filter for estimate of a state vector of essentially non-linear robots which have a complicated dynamics.*

Key words: *essentially non-linear systems, robots, unscented filter, unscented smoother, observer, estimation*

1. INTRODUCTION

One of main components of an adaptive robot control system is an observer. The observer is required to estimate unmeasured disturbances to their future rejection [1-8]. Measurements of external disturbances acting on a robot are almost impossible. The observer should contain a filter algorithm for noise reduction. In case of a rough terrain, soil parameters should be estimated. The approach, proposed in this paper, is based on evolution of an approach from paper [9] and integrating it with Unscented Kalman Filter from papers [2-3].

2. MATHEMATICAL MODEL OF MOBILE ROBOT

For an example, we will consider a mobile robot (MR) with differential drive structure. We represent the motion of MR in a fixed Cartesian coordinate system P. We link with the robot's center of gravity orthonormal basis Z. Size of Z is the same as P. A mathematical model of MR can be represented by the following system of vector-matrix equations [9-12]:

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$$\dot{Y} = \begin{bmatrix} P \\ \varphi \end{bmatrix} = RL_1 X + \eta_k \tag{1}$$

$$-L^T mL\dot{X} = M - F_s f' + L^{-T} F_O + F_N + \eta_d \tag{2}$$

$$F_s = \begin{bmatrix} R_g - R_c & 0 \\ 0 & R_g + R_c \end{bmatrix}$$

$$F_O = \begin{bmatrix} G\sin\alpha + R_{TR} + F_a \\ M_r \end{bmatrix}$$

$$R_g = (G/2)\cos\alpha\cos\beta$$

$$R_c = \varphi^2 Rh / gB$$

$$M_r = (\mu Gl\cos\alpha\cos\beta(1-(l\xi/2)^2)/4$$

where $\dot{Y}$ is the vehicle's position and orientation vector, X is vector of speeds of driven wheels, L and L_1 are matrixes of internal coordinates transformation, f' is vector of soil coefficients, α is pitch, β is roll, φ is yaw, m is matrix of mass and inertia parameters, M is driven moments on wheels, G is weight, g is constant g=9.8, ξ is shifting of turn poles, l is vehicle's length, B is vehicle's base, h is vehicle's mass height, M_r is a turn resistance in case a tracked vehicle, μ is a special coefficient, in general $\mu = f(f', B, r_c)$, where r_c is vehicle's turn radius, R_{TR} is trailer's resistance, F_a is air resistance, F_N is approximated external disturbance, as well as the influence of unmodelled dynamics in the dynamic model. There should be noted that the forces acting on MR are included in the equation of motion additive, which means the estimate of disturbances can actually correspond to real, $\eta_k \sim N(\eta_{mk}, \bar{R}_k)$ and $\eta_d \sim N(0, \bar{R}_d)$ are white Gaussian noises. We add a new variable $f = f' + f''$, such, that $f'' = F_s^{-1} F_N$. And proceed to the next transformation with (2):

$$-F_s^{-1} L^T mL\dot{X} = F_s^{-1} M - f + F_s^{-1} L^{-T} F_O \tag{3}$$

3. Nonlinear Observer

For estimation of the vector f, an approach proposed in [1] allows to estimate the unmeasurable disturbances without details of their structure. This approach draws on methods of designing nonlinear Luenberger observers, backstepping and disturbance approximation. Due to the fact that the vector f is additive in (3), it can be represented by the differential equation that provides zero order astaticism $\dot{f} = 0$:

$$\dot{X} = (F_s^{-1} L^T mL)^{-1} (-F_s^{-1} M - F_s^{-1} L^{-T} F_O + f) \tag{4}$$

Denote an estimation of the unmeasurable vector f by $\hat{f}$ and introduce a macro variable ε, which is equal to an estimation error. We define a differential equation for an asymptotic convergence of estimates and change variable $\hat{f}$:

$$\varepsilon = f - \hat{f} \tag{5}$$

$$\dot{\varepsilon} + A\varepsilon = 0 \tag{6}$$

$$\hat{f} = s(x, y) + \hat{z} \tag{7}$$

We differentiate (5) with (4, 6, 7) and get the following matrix equation of the state observer:

$$\dot{X} = (F_s^{-1} L^T mL)^{-1} (-F_s^{-1} (D_1 U - D_2 X) - F_s^{-1} L^{-T} F_O + AF_s^{-1} L^T mLX + \hat{z})$$

As described in [1] procedure, we defined independent of the unmeasurable variables function:

$s(x, y) = AF_s^{-1} L^T mLX$. So, the observer's equation gets as follows:

$$\hat{z}' = AF_s^{-1} M + AF_s^{-1} L^{-T} F_O - A^T AF_s^{-1} L^T mLX - \hat{z} \tag{8}$$

Without loss of generality, we represent power system and transmission as an inertial part $M = D_1 U - D_2 X$, then the dynamic equation becomes as follows:

$$\dot{X} = (F_s^{-1} L^T mL)^{-1} (-F_s^{-1} (D_1 U - D_2 X) - F_s^{-1} L^{-T} F_O + AF_s^{-1} L^T mLX + \hat{z}) \tag{9}$$

So, equation (9) with (1) and (7) becomes a complete differential system that describes the behavior of differential drives MR.

4. Noise Reduction

In addition to estimating the residual disturbances and dynamics we need to estimate sliding wheels on the soil and the angle of orientation of the orthonormal basis, linked with MR. These parameters received special attention because no vehicle can turn without slipping. Slipping is very significant in tracked vehicles. We suppose that the robot is equipped with accelerometer and three axes gyroscope. For the estimation of these parameters it is proposed to use a filtering algorithm based on unscented transformation (UT). This is caused by high noisiness, which depends on a sensor noise, vibration and a dynamic impact, which are certain to occur during motion of any vehicle in a rough terrain.

The first estimation method based on the UT was proposed in 1995 [2] and detailed in [3-5]. UT is the approximation of a state vector of variables by several sigma-points which are distributed around the main. As described in [3], assume x has mean $\bar{x}$ and covariance P_x . We form a matrix χ , whose size is 2L+1, and vectors χ_i as follows:

$$\chi_0 = \bar{x}$$

$$\chi_i = \bar{x} \pm \left(\sqrt{(L + \lambda) P_x}\right)_i, i = 1, \ldots, 2L \tag{10}$$

where λ – scaling parameter. Method of forming sigma-points defines their name, because the distribution depends on the square root of the covariance matrix, which is a generalization of the dispersion of a multidimensional random variable. UKF structure corresponds to the Kalman filter structure.

There it is possible to modify the UKF procedure described above by forming an augmented state variable, which concatenates the state and noise components together, so that the effect of the process and measurement noises can be used to better capture the odd-order moment information. If the noises are not additive the augmented version should produce more accurate estimate [4-5].

The prediction and update steps of UKF described as follows for augmented state variable $\tilde{x}$:

$$\tilde{x}_{k-1} = [x_{k-1}^T, q_{k-1}^T, r_{k-1}^T]^T$$

Then compute the predicted state mean and the predicted covariance

$$\hat{X}_k = f(X_{k-1}^x, X_{k-1}^q, k-1) \tag{11}$$
$$m_k^- = X_k^x w_m$$
$$P_k^- = X_k^x W [X_k^x]^T$$

where w_m and W are weight matrices, X_{k-1}^x and X_{k-1}^q are sigma-pints (10). By previous equation we denoted the components of sigma points which correspond to the actual state variables and process noise. The state transition function (11) is also augmented to incorporate the effect of process noise if necessary, which is now passed to the function as a second parameter.

The next step is update. There we compute the predicted mean and covariance of the measurement and the cross-covariance of the state and the measurement:

$$Y_k^- = h(\hat{X}_k, X_{k-1}^r, k) \tag{12}$$
$$\mu_k = Y_k^- w_m$$
$$S_k = Y_k^- W [Y_k^-]^T$$
$$C_k = \hat{X}_k W [Y_k^-]^T$$

where, μ_k, S_k are measure mean and covariance, respectively, C_k is cross-covariance of the state and measurement vectors. The measurement function (12) is now augmented to incorporate the effect of measurement noise, which is passed as a second parameter to the function. The third step will be described as follows:

$$K_k = C_k S_k^{-1}$$
$$m_k = m_k^- + K_k (y_k - \mu_k)$$
$$P_k = P_k^- - K_k S_k K_k^T$$

where K_k is the gain, m_k and P_k are filtered mean and covariance respectively.

The nonlinear system meant for filtering (11) is from (8, 9). Thus, the structure of the filter contains the nonlinear observer, constructed in accordance with the paper [1]. This approach allows combining two powerful techniques for ensuring an estimation convergence and filtering. Noteworthy, that similar approach has already been published in [6]. However, instead of a nonlinear observer, proposed in this paper, in [6] was used a sliding mode observer, which, strictly speaking, can only be applied to systems whose nonlinearity does not depend on the estimated vector, and therefore cannot be applied in an essentially non-linear and multilinked system, such as rough terrain MR.

UKF filter structure also satisfies the idea of extension of a phase space due to an extended estimation of the mean of noise. However, the state vector for dynamic model (9) is not augmented, because it is also extended by dynamic of the observer. We measure the following parameters:

$$\dot{P}, X, \alpha, \beta, \varphi \tag{13}$$

5. Smoothing a Robot's Path

During the movement of MR, besides estimation of its own state, robot can do other important tasks for itself, such as localization and mapping. Since the position and orientation of the robot attached to terrain description, the quality of description should be as exact as possible. Information about environment, defined by robot with significant errors, affects position and orientation errors, during the repeated movement in this area. To minimize this error we proposed using a smoothing filter. The main idea of a smoother is to run the filter back on the stored information about the state vector from the previous filter. One of the best methods of a smoothing algorithm is Unscented Rauch-Tung-Striebel smoother (URTSS) introduced in [4]. The URTSS can be used for computing a Gaussian approximation to the smoothing distribution of the step k:

$$\tilde{x}_{k-1} = [x_{k-1}^T, q_{k-1}^T]^T$$

Propagate the sigma points through the dynamic model:

$$\hat{X}_{k+1}^{-} = f(X_k^x, X_k^q, k)$$

Prediction step:

$$\begin{aligned} m_{k+1}^{-} &= \tilde{X}_{k+1}^{-x} w_m \\ P_{k+1}^{-} &= \tilde{X}_{k+1}^{-x} W [\tilde{X}_{k+1}^{-x}]^T \\ C_{k+1}^{-} &= \tilde{X}_{k+1}^{-x} W [\tilde{X}_k^x]^T \end{aligned}$$

where m_{k+1}^{-}, P_{k+1}^{-} and C_{k+1}^{-} are predicted mean, covariance and cross-covariance, respectively.

Third step will be described as follows:

$$\begin{aligned} D_k &= C_{k+1} [P_{k+1}^{-}]^{-1} \\ m_k^s &= m_k + D_k (m_{k+1}^s - m_{k+1}^{-}) \\ P_k^s &= P_k + D_k (P_{k+1}^s - P_{k+1}^{-}) D_k^T \end{aligned}$$

where D_k is the smoother gain, m_k^s and P_k^s are smoothed mean and covariance, respectively.

6. SIMULATION

Thus, estimation is carried out in three phases:

1. Measurement (13) and noise filtering in model (1);
2. Estimating the residual dynamics and unmeasured perturbation by (8, 9) and noise filtering;
3. Smoothing obtained in phase 2.

Phases 1-2 execute on each step, phase 3 executes the situation in one step. Simulation of the proposed algorithm of estimation MR, contains three tests. Simulation setup: model of differential drives robot with follows parameters: mass 700kg, l=2m, B=1.2m, $\mu = (f_l + f_r)/(3.4 + 0.6 r_t / B)$, $\xi = 0.5 l \tan\beta / \mu$, $R_{TR} = F_a = 0$, $\bar{R}_k = \bar{R}_d = diag(0.01)$, no cushioning. Root mean square error (RMSE) of all tests is presented in table 1. The first test is rising of soil coefficients. MR moving on strong line, speed is 20 m/s. Fig. 1 shows the estimation by UKF only.

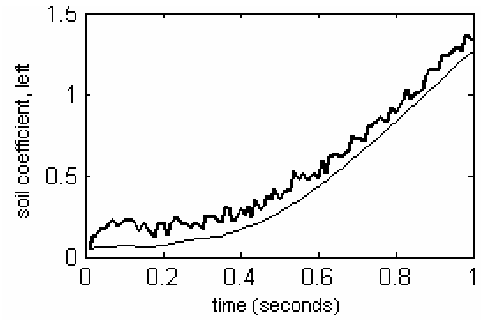

Fig. 1 Parameter f obtained through UKF (bold line) and real (thin line)

Figure 2 shows the estimation both by observer and UKF.

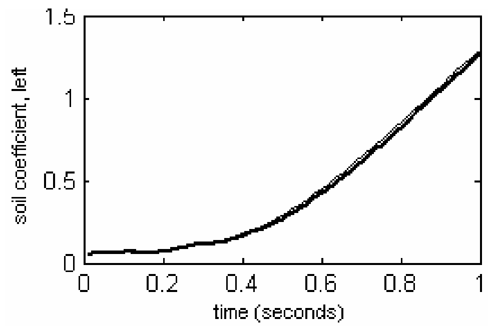

Fig. 2 Parameter f obtained through observer (bold line) and real (thin line)

The second test is run down the high hill at low speed, hull diameter is 6m, height is 2m. MR speed is 0.5m/s, soil coefficient f is 0.05. Figure 3 shows β filtered by UKF.

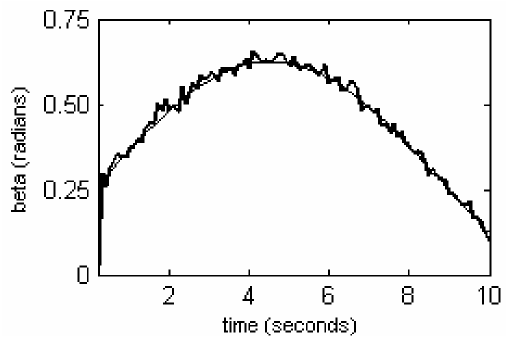

Fig 3 β obtained through UKF (bold line) and real (thin line)

And Fig. 4 shows β smoothed by URTSS.

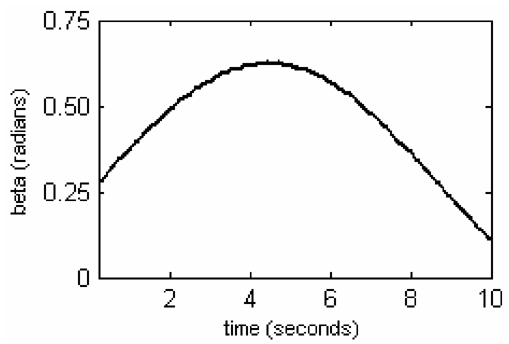

Fig 4 β obtained through URTSS (bold line) and real (thin line)

The third test is moving in a circle at high speed. Circle's radius is 15m, MR speed is 18 m/s, soil coefficient f is 0.05. Other tests are represented in Table 1.

Table 1 Computed RMSE for MR tests

Test	Parameter	UKF only	Observer and UKF
Rising soil coefficients	Soil coefficient, left	0.451	0.027
Run down the high hill at low speed	Roll angle	0.015	none
Run down the high hill at low speed	Soil coefficient, right	0.652	0.036
Moving in the circle	Slip coefficient, left	0.025	none
Moving in the circle	Soil coefficient, left	0.398	0.031

7. Conclusion

The results of simulation have shown a high speed and accuracy estimation. The approach introduced in this paper, based on nonlinear Luenberger's observer, backstepping and the integration with Unscented Kalman Filter for observation of the state vector and noise filtering of MR is universal, it could be used for a development of various types of highly manoeuvrable robots. Model of differential drive robot, introduced in this paper, can be used for synthesis of model of different types of such vehicles.

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